Mavzu:

Aniq integral.
Nyuton-Leybnits formulasi

REJA

- **Quyi va yuqori integral yig’indilar.**
- **Aniq integral.**
- **Aniq integralning asosiy xossalari.**
- **Aniq integralni hisoblash. Nyuton-Leybnits formulasi**

1. Quyi va yuqori integral yig'indilar.

Matematika, fizika, mexanika va boshqa fanlarda tadqiqotlar olib borishning eng yaxshi vositasi aniq integraldir. Egri chiziqlar bilan chegaralangan yuzalarni, yoylarning uzunliklarini, hajmlarni, ishni, tezlikni, yo'lni, inersiya momentlarini hisoblash aniq integralni hisoblashga keltiriladi.

$[a,b]$ kesmada uzluksiz $y=f(x)$ funksiya berilgan bo'lsin. m va M bilan shu oraliqdagi eng katta va eng kichik qiymatlarni belgilaymiz. $[a,b]$ kesmani

$$a=x_0,\ x_1,x_2,...,x_{n-1},x_n=b,$$

bo'lishini nuqtalari yordamida n ta qismlarga ajratamiz, bunda

$$x_0<x_1<x_2<...<x_n,$$

va

$$x_1-x_0=\Delta x_1,\ x_2-x_1=\Delta x_2,....,x_n-x_{n-1}=\Delta x_n$$

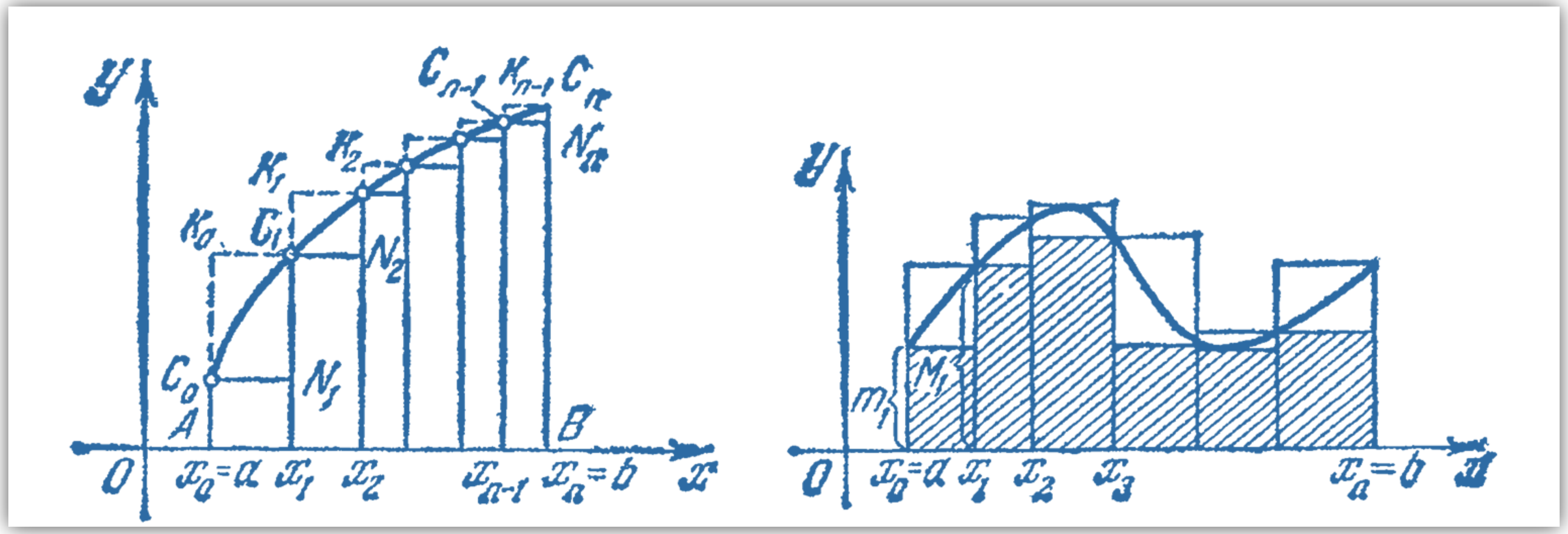
So'ngra, $y=f(x)$ funksiyaning eng katta va eng kichik qiymatlarini quyidagicha belgilaymiz

$$[x_0,x_1]\ m_1 \text{ va } M_1,$$

$$[x_1,x_2]\ m_2 \text{ va } M_2,$$

....

$$[x_{n-1},x_n]\ m_n \text{ va } M_n$$



y
C_{n-1}
K_{n-1}
C_n
N_n
K_2
K_1
K_0
C_1
N_2
C_0
N_1
A
B
O
$x_0=a$
x_1
x_2
x_{n-1}
$x_n=b$
x
y
M_1
m_1
O
$x_0=a$
x_1
x_2
x_3
$x_n=b$
x

Quyidagi yig'indilarni tuzamiz:

$$\underline{s_n} = m_1\Delta x_1 + m_2\Delta x_2 + ... + m_n\Delta x_n = \sum_{i=1}^{n} m_i \Delta x_i \qquad (1)$$

$$\overline{s_n} = M_1\Delta x_1 + M_2\Delta x_2 + ... + M_n\Delta x_n = \sum_{i=1}^{n} M_i \Delta x_i \qquad (2)$$

$\underline{s_n}$ - yig'indi quyi integral yig'indi, $\overline{s_n}$ -yig'indi esa yuqori integral yig'indi deb ataymiz.

Agar $f(x) \geq 0$ bo'lsa, u holda quyi integral yig'indi sonma-son $AC_0N_1C_1N_2...C_{n-1}N_nBA$ "ichki chizilgan zinasimon figura"ning yuzasiga teng, yuqori integral yig'indi sonma-son

$$AK_0C_1K_1...C_{n-1}K_{n-1}C_nBA$$

"tashqi chizilgan zinasimon figura"ning yuzasiga teng.

Quyi va yuqori integral yig'indilarning ba'zi xossalarini sanab o'tamiz:

a) $m_i \leq M_i$ bo'lganligi uchun $i(i = 1, 2, ..., n)$, (1) va (2) formulalar asosida topamiz

$$\underline{s_n} \leq \overline{s_n}.$$

(agar $f(x) = const$ bo'lsagina tenglik belgisi bo'ladi).

b)

$$m_1 \geq m, m_2 \geq m, ..., m_n \geq m,$$

bo'lganligi uchun, bu yerda m - $f(x)$ funksiyaning $[a,b]$dagi eng kichik qiymati,

$$\underline{s_n} = m_1\Delta x_1 + m_2\Delta x_2 + ... + m_n\Delta x_n \geq m\Delta x_1 + m\Delta x_2 + ... + m\Delta x_n =$$
$$= m(\Delta x_1 + \Delta x_2 + ... + \Delta x_n) = m(b - a)$$

Shunday qilib,

$$\underline{s}_n \geq m(b-a)$$

v)

$$M_1 \leq M, M_2 \leq M, ..., M_n \leq M,$$

bu yerda M - $f(x)$ funksiyaning $[a,b]$ dagi eng katta qiymati,

$$\overline{s_n} = M_1\Delta x_1 + M_2\Delta x_2 + ... + M_n\Delta x_n \leq M\Delta x_1 + M\Delta x_2 + ... + M\Delta x_n =$$
$$= M(\Delta x_1 + \Delta x_2 + ... + \Delta x_n) = M(b-a)$$

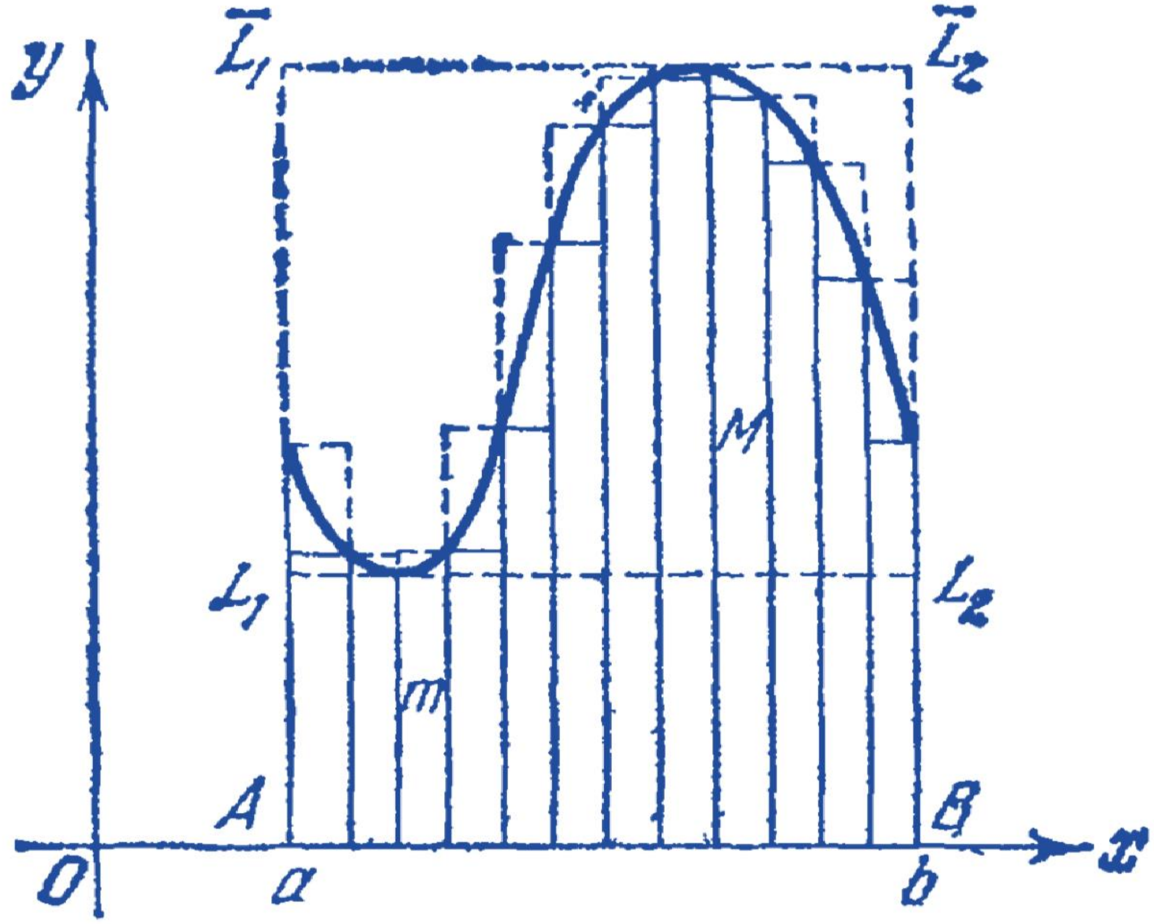
Shunday qilib,

$$\overline{s_n} \leq M(b-a)$$

Olingan tengsizliklarni birlashtirib, topamiz

$$m(b-a) \leq \underline{s}_n \leq \overline{s_n} \leq M(b-a)$$

Agar $f(x) \geq 0$ bo'lsa, u holda oxirgi tengsizlik sodda geometrik ma'noga ega, chunki $m(b-a)$ va $M(b-a)$ ko'paytmalar mos ravishda "ichki chizilgan" AL_1L_2B va "tashqi chizilgan" $A\overline{L_1}\overline{L_2}B$ to'gri to'rtburchaklarning yuzalariga teng.



y
$\bar{L}_1$
$\bar{L}_2$
M
L_1
L_2
m
A
B
O
a
b
x

2. Aniq integral

Bundan avvalgi paragrafdagi masalani o'rganishda davom etamiz. $[x_0, x_1],[x_1, x_2],...,[x_{n-1}, x_n]$ kesmalardan har birida bittadan nuqta olib, ularni $\xi_1, \xi_2, ..., \xi_n$ bilan belgilaymiz (209-rasm),

$$x_0 < \xi_1 < x_1, x_1 < \xi_2 < x_2, x_{n-1} < \xi_n < x_n$$

Bu nuqtalarning har birida $f(\xi_1), f(\xi_2),..., f(\xi_n)$ funksiyaning qiymatlarini hisoblaymiz. Endi

$$s_n = f(\xi_1)\Delta x_1 + f(\xi_2)\Delta x_2 + ... + f(\xi_n)\Delta x_n = \sum_{i=1}^{n} f(\xi_i)\Delta x_i \qquad (1)$$

yig'indini tuzamiz. Bu yig'indi $f(x)$ funksiyaning $[a,b]$ kesmadagi integral yig'indisi deyiladi. Ixtiyoriy ξ_i nuqta $[x_{i-1}, x_i]$ kesmaga tegishli bo'lganda

$$m_i \le f(\xi_i) \le M_i$$

va barcha $\Delta x_i > 0$ bo'lganligi uchun

$$m_i \Delta x_i \le f(\xi_i)\Delta x_i \le M_i \Delta x_i$$

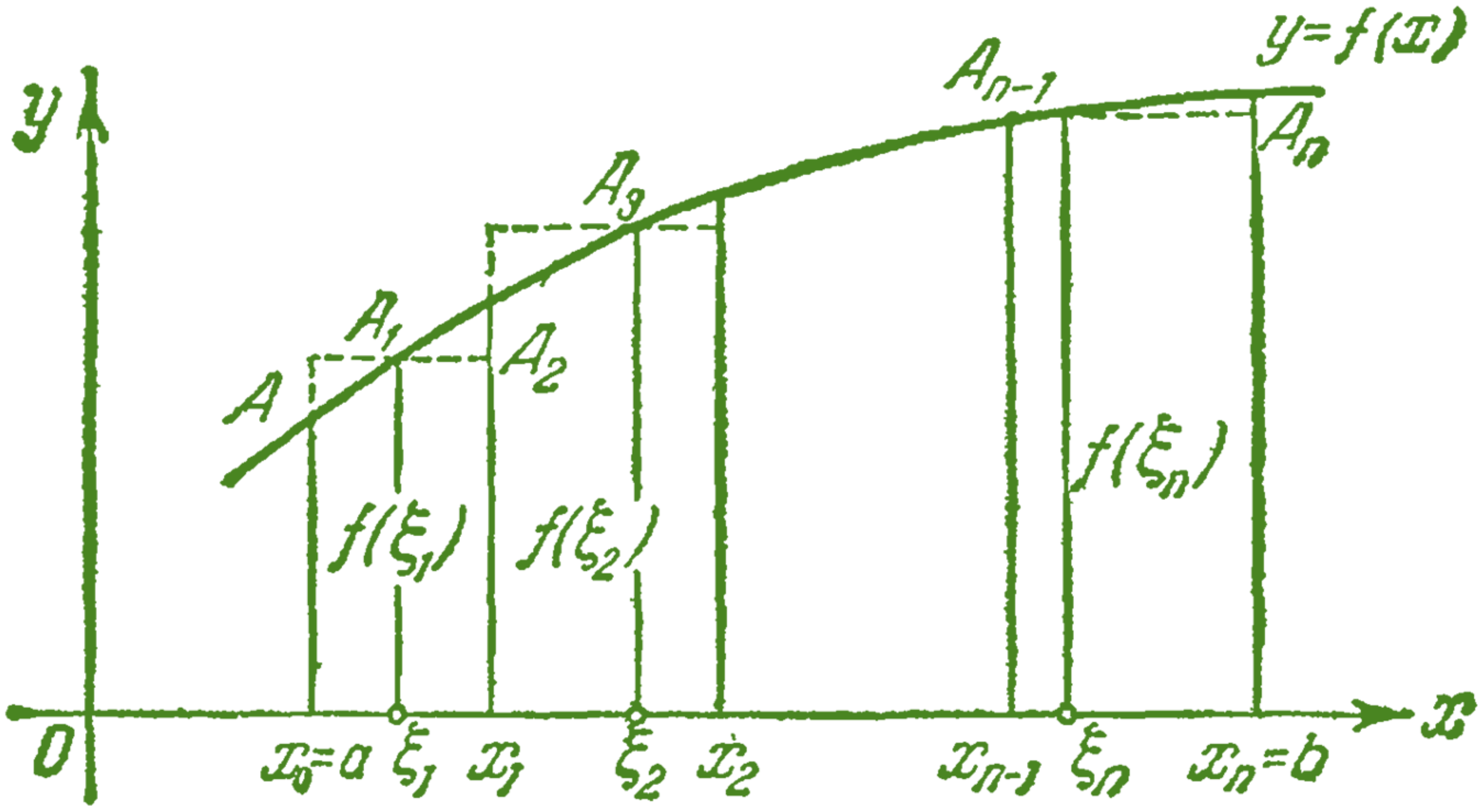
demak,

$$\sum_{i=1}^{n} m_i \Delta x_i \le \sum_{i=1}^{n} f(\xi_i)\Delta x_i \le \sum_{i=1}^{n} M_i \Delta x_i$$

yoki

$$\underline{s_n} \le s_n \le \overline{s_n} \qquad (2)$$

Oxirgi tengsizlikning ma'nosi shuki, $f(x) \ge 0$ bo'lganda yuzasi s_n ga teng bo'lgan yuzani chegaralovchi siniq chiziq "ichki chizilgan" va "tashqi chizilgan" siniq chiziqlar orasida joylashgan.



y
y=f(x)
A
A1
A2
A3
An-1
An
f(ξ1)
f(ξ2)
f(ξn)
x
0
x0=a
ξ1
x1
ξ2
x2
xn-1
ξn
xn=b

s_n yig'indi $[a,b]$ kesmani $[x_{i-1},x_i]$ kesmalrga bo'lish usuliga va shu kesmalar ichida ξ_i nuqtalarning tanlanishiga bog'liq.

Endi $\max[x_{i-1},x_i]$ bilan $[x_0,x_1],[x_1,x_2],...,[x_{n-1},x_n]$ kesmalar uzunliklaridan eng kattasini belgilaymiz. $[a,b]$ kesma $[x_{i-1},x_i]$ kesmalarga shunday bo'lamizki, $\max[x_{i-1},x_i]\to 0$ bo'lsin. Albatta, bunda, kesmalar soni n cheksizlikka intiladi. Har bir bo'lish uchun tegishli ξ_i qiymatlarni tanlab

$$\sum_{i=1}^{n} f(\xi_i)\Delta x_i$$

integral yig'indini tuzish mumkin. Shunday qilib, bo'linishlar ketma-ketligi va unga mos integral yig'indilar ketma-ketligi haqida gapirish mumkin. Shunday bir ketma-ketlikni tanlasakki, $\max \Delta x_i \to 0$ bo'lsa, u holda yig'indi I limitga intilsin.

Agar $[a,b]$ kesmani $\max \Delta x_i \to 0$ bo'ladigan qilib bo'lganda va ξ_i nuqtalar ixtiyoriy tanlashganda $\sum_{i=1}^{n} f(\xi_i)\Delta x_i$ yig'indi o'sha I limitga intilsa, u holda $f(x)$ - integral osti funksiya - $[a,b]$ kesmada integrallanuvchi, I limit esa $[a,b]$ kesmada aniqlangan $f(x)$ funksiyaning aniq integrali deyiladi. Uni $\int_a^b f(x)dx$ deb belgilaymiz va

$$\lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(\xi_i)\Delta x_i = \int_a^b f(x)dx$$

a soni integralning quyi limiti, b - yuqori limiti deyiladi. $[a,b]$ kesma integrallash kesmasi, x esa integrallash o'zgaruvchisi deyiladi.

Agar $y=f(x)$ funksiya $[a,b]$ kesmada uzluksiz bo'lsa, u holda u kesmada integrallanuvchidir.

Albatta, agar $\Delta x_1 \to 0$ bo'ladigan qandaydir bo'linishlar ketma-ketligida $f(x)$ funksiya $\underline{s_n}$ va $\overline{s_n}$ integral yig'indilarni qarasak, u holda bu yig'indilar I limitga - $f(x)$ funksiyadan olingan aniq integralga intiladi:

$$\lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} m_i \Delta x_i = \int_a^b f(x)dx$$

$$\lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} M_i \Delta x_i = \int_a^b f(x)dx$$

Uzulishli funksiyalar orasida integrallanadigan funksiyalar ham, integrallanmaydigan funksiyalar ham bor.

Agar $y = f(x)$ integral osti funksiyaning grafigini qursak, u holda $f(x) \geq 0$ bo'lganda

$$\int_a^b f(x)dx$$

integral son jihatdan ko'rsatilgan egri chiziq $x = a, x = b$ to'g'ri chiziqlar va Ox o'q bilan chegaralangan egri chiziqli trapetsiyaning yuziga teng.

Shuning uchun, agar $y = f(x)$ egri chiziq $x = a, x = b$ to'g'ri chiziqlar va Ox o'q bilan chegaralangan egri chiziqli trapetsiyaning yuzasini hisoblash kerak bo'lsa, u holda bu Q yuza

$$Q = \int_a^b f(x)dx \qquad (3)$$

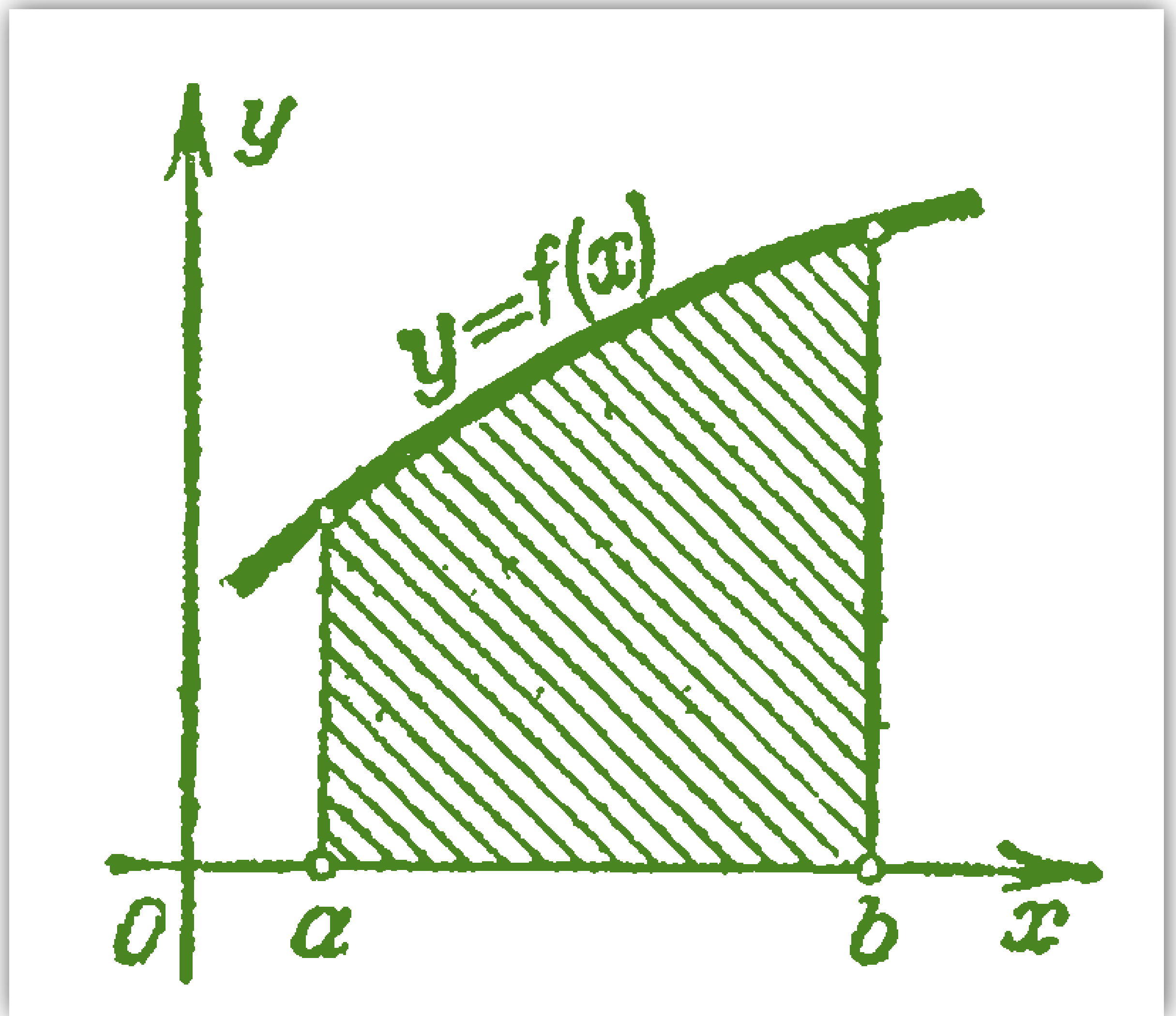
formula bilan hisoblanadi.

Izox 1. Shuni alohida ta'kidlash kerakki, aniq integral faqat $f(x)$ funksiyaning ko'rinishiga va integrallash chegaralariga bog'liq, lekin integral o'zgaruvchisiga bog'liq emas. Shuning uchun aniq integralning qiymatini o'zgartirmagan holda x harfining o'rniga ixtiyoriy boshqa xarfni olishimiz mukin:

$$\int_a^b f(x)dx = \int_a^b f(t)dt = ... = \int_a^b f(z)dz$$

Aniq integral tushunchasini kiritayotganda bu $a < b$ deb faraz qildik. $b < a$ bo'lgan holda ta'rifga ko'ra

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$


y
y=f(x)
0
a
b
x

Masalan,

$$\int_5^0 x^2dx = -\int_0^5 x^2dx$$

Endi $a = b$ bo'lganda ta'rifga ko'ra, ixtiyoriy $f(x)$ funksiya uchun

$$\int_a^a f(x)dx = 0 \qquad (5)$$

tenglik o'rinli.

Bu geometrik nuqtai nazardan ham tabiiy. Haqiqatan ham egri chiziqli trapetsiya asosi nolga teng uzunlikka ega, demak, uning yuzasi nolga teng.

3. Aniq integralning asosiy xossalari

1-xossa. O'zgarmas ko'paytuvchini aniq integral belgisidan tashqariga chiqarish mumkin: agar $A = const$ bo'lsa, u holda

$$\int_a^b Af(x)dx = A\int_a^b f(x)dx \qquad (1)$$

Isboti.

$$\int_a^b Af(x)dx = \lim_{\max \Delta x \to 0} \sum_{i=1}^{n} Af(\xi_i)\Delta x_i =$$

$$= A \lim_{\max \Delta x \to 0} \sum_{i=1}^{n} f(\xi_i)\Delta x_i A\int_a^b f(x)dx$$

2-xossa. Bir necha funksiyalarning algebraic yig'indisidan olingan aniq integral qo'shiluvchilardan olingan integrallarning algebraic yig'indisiga teng. Ikki qo'shiluvchi bo'lgan holda

$$\int_a^b [f_1(x) + f_2(x)]dx = \int_a^b f_1(x)dx + \int_a^b f_2(x)dx \qquad (2)$$

Isboti.

$$\int_a^b [f_1(x) + f_2(x)]dx = \lim_{\max \Delta x \to 0} \sum_{i=1}^{n} [f_1(\xi_i) + f_2(\xi_i)]\Delta x_i =$$

$$= \lim_{\max \Delta x \to 0} \left[\sum_{i=1}^{n} f_1(\xi_i)\Delta x_i + \sum_{i=1}^{n} f_2(\xi_i)\Delta x_i \right] =$$

$$= \lim_{\max \Delta x \to 0} \sum_{i=1}^{n} f_1(\xi_i)\Delta x_i + \lim_{\max \Delta x \to 0} \sum_{i=1}^{n} f_2(\xi_i)\Delta x_i =$$

$$= \int_a^b f_1(x)dx + \int_a^b f_2(x)dx$$

Qo’shiluvchi soni ixtiyoriy bo’lganda ham shunaqa isbotlanadi.

1- va 2- xossalar $a<b$ hol uchun isbotlangan bo’lsada, ular $a\geq b$ holda ham o’rinli.

Ammo quyidagi xossa faqat $a<b$ holda o’rinli:

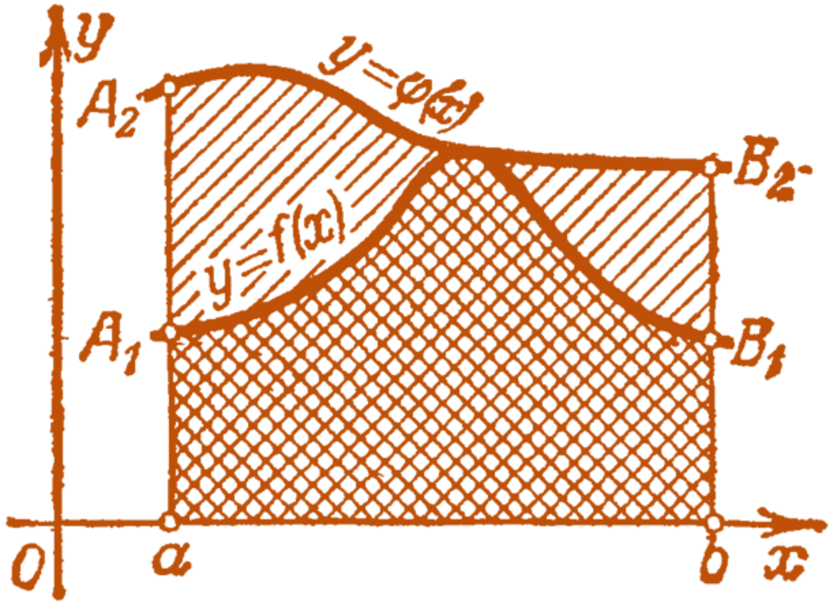
3-xossa. Agar $[a,b]$ kesmada ($a<b$) $f(x)$ va $\varphi(x)$ funksiyalar $f(x)\leq\varphi(x)$ shartni qanoatlantirsa, u holda

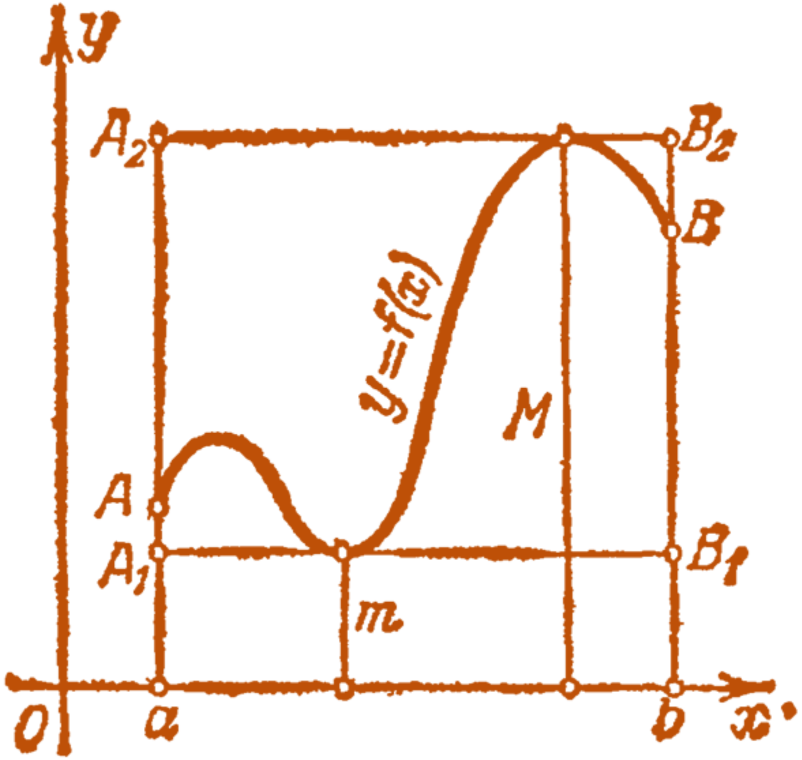
$$\int_a^b f(x)dx \leq \int_a^b \varphi(x)dx \qquad (3)$$

Isboti. Quyidagi ayirmani qaraymiz:

$$\int_a^b \varphi(x)dx - \int_a^b f(x)dx = \int_a^b [\varphi(x) - f(x)]dx =$$

$$= \lim_{\max \Delta x \to 0} \sum_{i=1}^{n} [\varphi(\xi_i) - f(\xi_i)]\Delta x_i$$


y
A2
y=φ(x)
B2
y=f(x)
A1
B1
0
a
b
x


y
A2
B2
B
y=f(x)
M
A
A1
B1
m
0
a
b
x

Bu yerda har bir ayirma, $\varphi(\xi_i) - f(\xi_i) \geq 0, \Delta x_i \geq 0$. Demak, yig'indining har bir qo'shiluvchisi nomanfiy, butun yig'indi ham nomanfiy va uning limiti ham nomanfiy, ya'ni

$$\int_a^b [\varphi(x) - f(x)]dx \geq 0$$

yoki

$$\int_a^b \varphi(x)dx - \int_a^b f(x)dx \geq 0$$

bu yerdan (3) tengsizlik kelib chiqadi.

Agar $f(x) > 0$ va $\varphi(x) > 0$ bo'lsa, aytib o'tilgan xossa geometric ma'noga ega (213-rasm). $\varphi(x) \geq f(x)$ bo'lganligi uchun aA_1B_1b egri chiziqli trapetsiyaning yuzasi aA_2B_2b egri chiziqli trapetsiya yuzasidan kata emas.

4-xossa. Agar m va M - $f(x)$ funksiyaning $[a,b]$ kesmadagi eng kichik va eng kata qiymatlari bo'lib, $a \leq b$ bo'lsa, u holda

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a) \qquad (4)$$

Isbot. Shartga ko'ra

$$m \leq f(x) \leq M$$

(3) xossa asosida topamiz:

$$\int_a^b m dx \leq \int_a^b f(x)dx \leq \int_a^b M\,dx \qquad (4')$$

Ammo

$$\int_a^b m dx = m(b-a) \qquad \int_a^b M\,dx = M(b-a)$$

Bu ifodalarni (4') tengsizlikka qo'shib (4) tengsizlikni olamiz.

Agar $f(x) \geq 0$ bo'lsa, u holda bu xossa geometric talqinga ega (214-rasm): $aABb$ egri chiziqli trapetsiyaning yuzi aA_1B_1b va aA_2B_2b to'g'ri to'rtburchaklarning yuzalari orasida joylashgan.

5-xossa. (o'rta qiymat haqida teorema). Agar $f(x)$ funksiya $[a,b]$ kesmada uzluksiz bo'lsa, u holda bu kesmada shunday ξ nuqta topiladiki,

$$\int_a^b f(x)dx = (b-a)f(\xi) \qquad (5)$$

tenglik o'rinli bo'ladi.

Isbot. Aniqlik uchun $a<b$ bo'lsin. Agar m va M mos ravishda $f(x)$ ning $[a,b]$ kesmadagi eng kichik va eng kata qiymatlari bo'lsa, u holda (4) formulaga binoan

$$m \leq \frac{1}{b-a}\int_a^b f(x)dx \leq M$$

Bu yerdan

$$\frac{1}{b-a}\int_a^b f(x)dx = \mu$$, bu yerda $m \le \mu \le M$

$f(x)$ funksiya uzluksiz bo'lganligi uchun, u m va M orasidagi hamma qiymatlarni qabul qiladi. Demak, $\xi(a \le \xi \le b)$ biror qiymatida $\mu = f(\xi)$ bo'ladi, ya'ni

$$\int_a^b f(x)dx = f(\xi)(b-a)$$

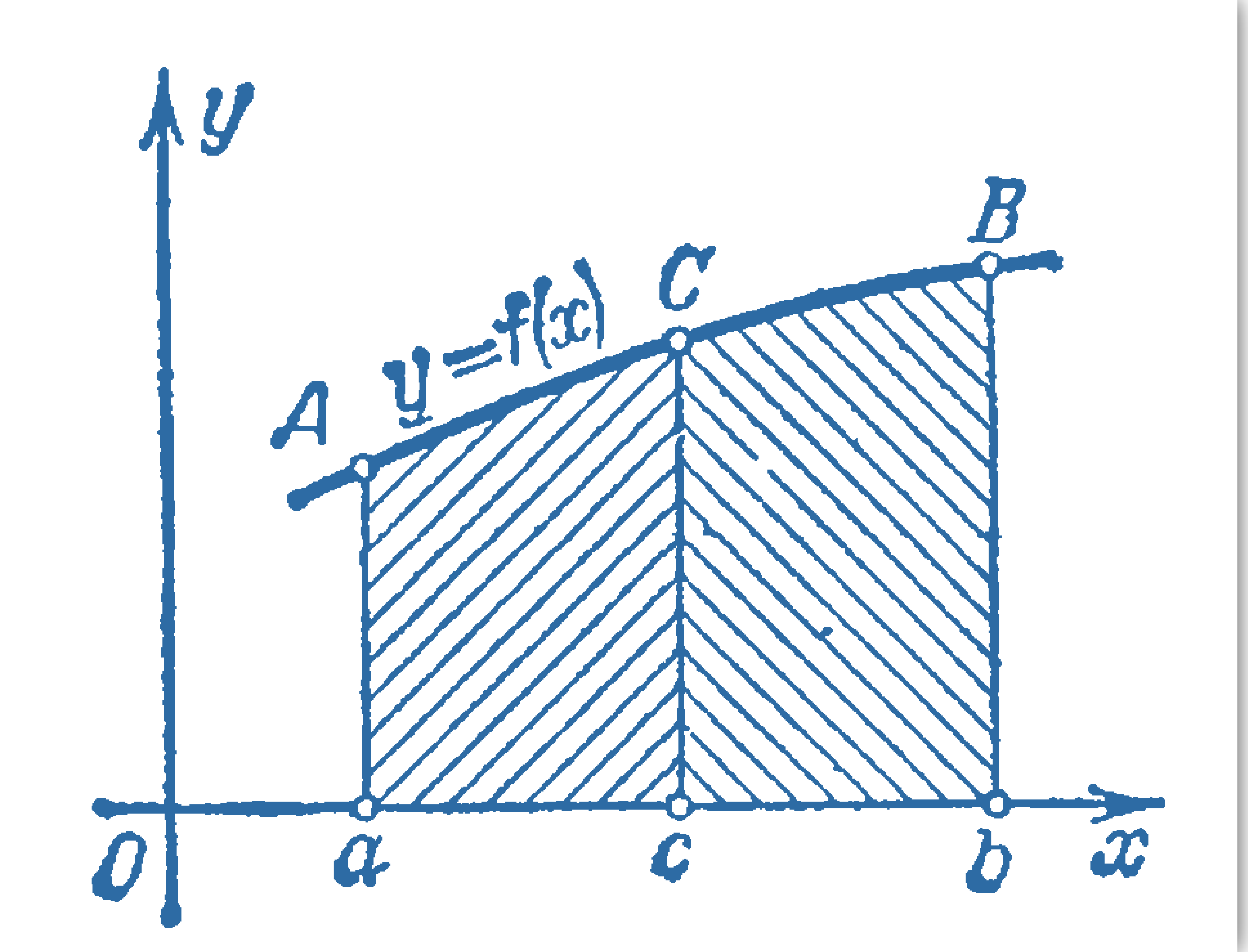
6-xossa. Ixtiyoriy uchta a,b,c sonlar uchun

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \qquad (6)$$

Isbot. $a < c < b$ deb faraz qilamiz va $f(x)$ funksiya uchun $[a,b]$ kesmada integral yig'indisini topamiz.

Integral yig'indining limiti $[a,b]$ kesmani bo'laklarga bo'lish usuliga bog'liq emas, shuning uchun biz $[a,b]$ kesmalarga shunday ajratamizki, c nuqta bo'linish nuqtasi bo'lsin. So'ngra $\sum_a^b$ integral yig'indini ikkita yig'indilarga ajratamiz:

$\sum_a^c$ va $\sum_c^b$


y
B
C
y=f(x)
A
O
a
c
b
x

U holda

$$\sum_a^b f(\xi_i)\Delta x_i = \sum_a^c f(\xi_i)\Delta x_i + \sum_c^b f(\xi_i)\Delta x_i$$

Oxirgi tenglikda $\max\Delta x_i \to 0$ bo'lganda limitga o'tib (6) munosabatni olamiz.

Agar $a<b<c$ bo'lsa, isbotlanganlar asosida yozamiz:

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$

yoki

$$\int_a^b f(x)dx = \int_a^c f(x)dx - \int_b^c f(x)dx$$

Ammo 2-§dagi (4) formulaga asosan

$$\int_b^c f(x)dx = -\int_c^b f(x)dx$$

Shuning uchun

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

215-rasmda 6-xossaning $f(x)>0$, $a<c<b$ bo'lganda geometric talqini aks ettirilgan: $aABb$ trapetsiyaning yuzi $aACc$ va $cCBb$ trapetsiyalar yuzalarining yig'indisiga teng.

4. Aniq integralni hisoblash. Nyuton-Leybnits formulasi

$$\int_a^b f(x)dx$$

Aniq integralda quyi a chegara mahkamlangan, yuqori b chegara esa o'zgraib tursin. U holda integralning qiymati ham o'zgarib turadi, ya'ni integral yuqori chegaraning funksiyasi bo'lib qoladi.

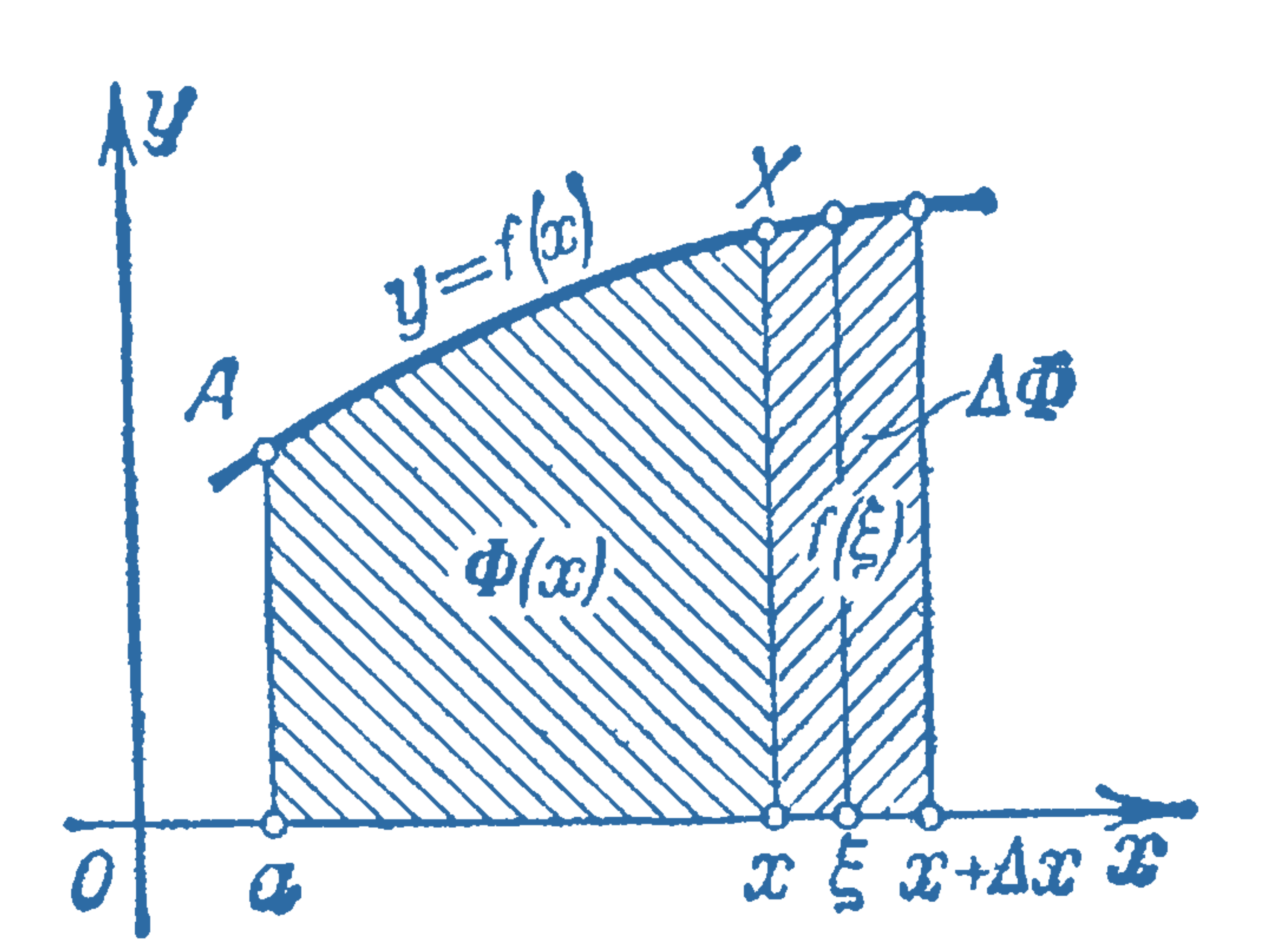
Yuqori chegarani x bilan, integral o'zgaruvchini t bilan belgilaymiz:

$$\int_a^x f(t)dt$$

Integralga ega bo'lamiz. a o'zgarmas son bo'lganda bu integral x yuqori chegaraning funksiyasi bo'ladi. Bu funksiyani biz $\Phi(x)$ bilan belgilaymiz:

$$\Phi(x) = \int_a^x f(t)dt$$

Agar $f(t)$ - nomanfiy funksiya bo'lsa, u holda $\Phi(x)$ miqdor son jihatdan $aAXx$ egri chiziqli trapetsiyaning yuziga teng (216-rasm). x o'zgarganda bu yuza o'zgarishi ochiq ravshan.



y
y=f(x)
Y
A
ΔΦ
Φ(x)
f(ξ)
0
a
x
ξ
x+Δx
x

$\Phi(x)$ funksiyaning hosilasini topamiz, ya'ni (1) integraldan yuqori chegara bo'yicha hosila olamiz.

Teorema 1. Agar $f(x)$ uzluksiz funksiya va $\Phi(x)=\int_a^x f(t)dt$ bo'lsa, u holda

$$\Phi'(x)=f(x)$$

tenglik o'rinli.

Boshqacha aytganda, aniq integraldan yuqori chegara bo'yicha olingan hosila integral ostidagi funksiyaga teng.

Isbot. x argumentga musbat yoki manfiy Δx orttirma beramiz; u holda topamiz (6-xossa):

$$\Phi(x+\Delta x)=\int_a^{x+\Delta x} f(t)dt=\int_a^x f(t)dt+\int_x^{x+\Delta x} f(t)dt$$

$\Phi(x)$ funksiyaning orttirmasi

$$\Delta\Phi=\Phi(x+\Delta x)-\Phi(x)=\int_a^x f(t)dt+\int_x^{x+\Delta x} f(t)dt-\int_a^x f(t)dt$$

ga teng, ya'ni

$$\Delta\Phi=\int_x^{x+\Delta x} f(t)dt$$

Oxirgi integralga o’rta qiymat haqidagi teoramani qo’llaymiz (5-xossa).

$$\Delta\Phi = f(\xi)(x+\Delta x - x) = f(\xi)\Delta x$$

bu yerda ξ miqdor x va $x+\Delta x$ orasida joylashgan.

Funksiya orttirmasining argument orttirmasiga nisbatan topamiz:

$$\frac{\Delta\Phi}{\Delta x} = \frac{f(\xi)\Delta x}{\Delta x} = f(\xi)$$

Demak,

$$\Phi'(x) = \lim_{\Delta x\to 0}\frac{\Delta\Phi}{\Delta x} = \lim_{\Delta x\to 0} f(\xi)$$

Ammo $\xi \to x$ da $\Delta x \to 0$ bo’lganligi uchun

$$\lim_{\Delta x\to 0} f(\xi) = \lim_{\xi\to x} f(\xi)$$

$f(x)$ uzliksiz bo’lganligi uchun

$$\lim_{\xi\to x} f(\xi) = f(x)$$

Shunday qilib, $\Phi'(x) = f(x)$. Teorema isbotlandi.

Teorema 2. Agar $F(x)$ funksiya $f(x)$ uzluksiz funksiyaning qandaydir boshlang'ichi bo'lsa, u holda

$$\int_a^b f(x)dx = F(b) - F(a) \qquad (2)$$

formula o'rinli. Bu formula Nyuton-Leybnits formulasi deyiladi.

Isbot. $F(x)$ funksiya $f(x)$ funksiyaning biror boshlang'ichi bo'lsin. 1-teoremaga ko'ra, $\int_a^x f(t)dt$ funksiya ham $f(x)$ funksiyaning boshlang'ichi bo'ladi. Ammo berilgan funksiyaning ixtiyoriy ikkita boshlang'ichlari bir biridan o'zgarmas C^* qo'shiluvchiga farq qiladi.

$$\int_a^x f(t)dt = F(x) + C^* \qquad (3)$$

Endi C^* o'zgarmasni toppish uchun bu tenglikda $x = a$ deb olamiz, u holda

$$\int_a^a f(t)dt = F(a) + C^*$$

yoki

$$0 = F(a) + C^*$$

Bu yerdan $C^* = -F(a)$. Demak,

$$\int_a^x f(t)dt = F(x) - F(a)$$

Endi $x = b$ deb olib, Nyuton-Leybnits formulasini olamiz:

$$\int_a^b f(t)dt = F(b) - F(a)$$

yoki integrallash o'rniga x ni qo'ysak

$$\int_a^b f(x)dx = F(b) - F(a)$$

$F(b) - F(a)$ ayirma F boshlang'ich funksiyaning tanlanishiga bog'liq emas.

Agar

$$F(b) - F(a) = F(x)\Big|_a^b$$

belgilash kiritsak

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

Nyuton-Leybnits formula aniq integrallarni hisoblashning qulay usulidir.