mavzu. Boshlang’ich funksiya. Aniqmas integral. Differensial tenglamalar haqida tushuncha.

Asosiy tushunchalar

- Boshlang'ich funktsiya tushunchasi.
- Aniqmas integral, uning xossalari.
- Asosiy integrallash jadvali.
- Aniq integral, uning geometrik ma'nosi.
- Aniq integralning asosiy xossalari.
- Nyuton-Leybnits formulasi.
- Aniq integralning tadbiqlari.

B.B.B. jadvali

T/r	Bilaman	Bilmayman	Bilib oldim
1	Boshlang'ich funktsiya		
2	Aniqmas integral		
3	Aniq integral		
4	Aniq integralning geometrik ma'nosi		
5	Aniq integralning xossalari		
6	Nyuton-Leybnits formulasi		

Ta'rif. Agar *[a,b]* kesmada aniqlangan *f(x)* funksiya uchun bu kesmaning barcha nuqtalarida $F^1(x)=f(x)$ tenglik bajarilsa, *F(x)* funksiya shu kesmada *f(x)* funksiyaga nisbatan boshlang`ich funksiya deb ataladi.

Ta'rif. Agar *F(x)* funksiya biror oraliqda *f(x)* funksiyaning boshlang'ich funksiyasi bo'lsa,

u holda *F(x)+C* (bu yerda *C* – ihtiyoriy doimiy) funksiyalar to'plami shu kesmada *f(x)*

funksiyaning aniqmas integrali deyiladi va quyidagicha belgilanadi:

$$\int f(x)dx = F(x) + C$$

Misol:

$$\int \cos x dx = \sin x + C$$ chunki

$$(\sin x)' = \cos x$$

Aniqmas integralning xossalari:

1) Aniqmas integralning hosilasi integral ostidagi funksiyaga teng, ya’ni

$$\left(\int f(x)dx\right)' = f(x)$$

2) Aniqmas integralning differensiali integral belgisi ostidagi ifodaga teng, ya’ni

$$d\left(\int f(x)dx\right) = f(x)dx$$

3) Biror funksiyaning hosilasidan olingan aniqmas integral shu funksiya bilan ihtiyoriy o‘zgarmasning yig‘indisiga teng, ya’ni

$$\int F'(x)dx = F(x) + C$$

4) Biror funksiyaning differentsialidan olingan aniqmas integral shu funksiya bilan ihtiyoriy o‘zgarmasning yig‘indisiga teng, ya’ni

$$\int dF(x) = F(x) + C$$

5) Chekli sondagi funksiyalarning algerbaik yig‘indisidan olingan aniqmas integral shu funksiyalarning har biridan olingan aniqmas integrallarning algebraik yig‘indisiga teng, ya’ni

$$\int \left(f_1(x)+f_2(x)+f_3(x)\right)dx=\int f_1(x)dx+\int f_2(x)dx+\int f_3(x)dx$$

Amaliy mashg'ulot rejasi

1. Boshlang‘ich funksiyani topishga doir misollar.
2. Aniqmas integralni hisoblashga doir misollar.

Integrallash jadvali asosida aniq integralni topish va hisoblash.

Aniq integralni Nyuton-Leybnits formulasi yordamida hisoblash.

T/r	Mavzu savoli	Bilaman "+" Bilmayman "-"	Bildim "+" Bila olmadim "-"
1	2	3	4
1	Boshlang'ich funksiyani topishga doir misollar yechish		
2	Aniqmas integralni hisoblashga doir misollar yechish.		
3	Aniq integralni hisoblashda foydalaniladigan asosiy formulalar		
4	Aniq integralni hisoblashga doir misollar yechish		

Savollar:

1. Boshlang‘ich funksiya deb nimaga aytiladi?

2. Aniqmas integralni hisoblash qoidalari qanday?

Misollar:

1. If $\int cosu\, du = sinu + C$, then to find $\int \cos(ax)\, dx$ one can scale the integral by letting $u = ax \; with \; du = adx$ to obtain

   $\frac{1}{a}\int cosu\, du = \frac{1}{a} sinu + C = \frac{1}{a}\sin(ax) + C$

Adabiyot:J.H.Heinbockel. Introduction to Calculus Volume 1, p.184, example 3-4

Agar $\int cosu\, du = sinu + C$ bo'lsa, u holda $\int \cos(ax)\, dx$ integralni topish uchun quyidagi belgilashni kiritamiz: $u = ax, du = adx$; u holda

$$\frac{1}{a}\int cosu\, du = \frac{1}{a} sinu + C = \frac{1}{a}\sin(ax) + C$$

bo'ladi.

1. Evaluate the integral $I = \int \frac{11x-43}{x^2-6x+5} dx$.

Solution

Here the integrand $\frac{11x-43}{x^2-6x+5}$ is a rational function with the degree of the numerator less than the degree of the denominator. Observe that the denominator has linear factors and so one can write

$$f(x) = \frac{11x-43}{x^2-6x+5} = \frac{11x-43}{(x-1)(x-5)} = \frac{A_1}{(x-1)} + \frac{A_2}{(x-5)} \quad (1)$$

where A1,A2 are constants to be determined. Multiply both sides of equation(1) by the factor (x − 1) and show

Adabiyot:J.H.Heinbockel. Introduction to Calculus Volume 1, p.196, example 3-13

Quyidagi $I = \int \frac{11x-43}{x^2-6x+5} dx$ integralni hisoblang.

Yechish:

Integral ostidagi funktsiya $\frac{11x-43}{x^2-6x+5}$ ratsional funktsiya bo’lib, bu ifoda suratining darajasi maxrajdagi ifoda darajasidan kichikdir. Maxrajdagi ifoda chiziqli bo’lgani uchun quyidagini yozishimiz mumkin:

$$f(x) = \frac{11x-43}{x^2-6x+5} = \frac{11x-43}{(x-1)(x-5)} = \frac{A_1}{(x-1)} + \frac{A_2}{(x-5)} \quad (1)$$

$$\frac{11x-43}{(x-5)} = A_1 + \frac{A_2(x-1)}{(x-5)}.$$

Evaluate equation using the value x = 1 to show A1 = 8. Next multiply equation on both sides by the other factor (x – 5) and show

Bu yerda A_1 va A_2 lar aniqlangan o'zgarmaslardir. Tenglikning ikkala tomonini $(x-1)$ ifodaga ko'paytiramiz:

$$\frac{11x-43}{(x-5)} = A_1 + \frac{A_2(x-1)}{(x-5)}.$$

Tenglikda $x = 1$ deb olsak, $A_1 = 8$ bo'ladi. Endi (1) tenglikning ikkala tomonini (x-5) ga ko'paytiramiz va

$$\frac{11x-43}{(x-1)} = \frac{A_1(x-5)}{(x-1)} + A_2$$

Evaluate the equation using the value x = 5 to show A_2 = 3. One can then write

$$I = \int \frac{11x-43}{x^2-6x+5}dx = \int \left[\frac{8}{x-1} + \frac{3}{x-5}\right]dx = 8\int \frac{dx}{x-1} + 3\int \frac{dx}{x-5}$$

$\frac{11x-43}{(x-1)} = \frac{A_1(x-5)}{(x-1)} + A_2$ ga ega bo'lamiz.

Oxirgi tenglikda $x = 5$ deb olsak, $A_2 = 3$ bo'ladi. Demak,

$$I = \int \frac{11x-43}{x^2-6x+5}dx = \int \left[\frac{8}{x-1} + \frac{3}{x-5}\right]dx = 8\int \frac{dx}{x-1} + 3\int \frac{dx}{x-5}$$

Both integrals on the right-hand side of this equation are of the form $\int \frac{du}{u}$ and consequently one finds

$$I = 8ln|x-1| + 3ln|x-5| + C$$

where C is a constant of integration. Observe that C is an arbitrary constant and so one can replace C by lnK, to make the algebra easier, where K > 0 is also an arbitrary constant. This is done so that all the terms in the solution will be logarithm terms
and therefore can be combined.

Tenglikning o'ng tomonidagi ikkala integral $\int \frac{du}{u}$ ko'rinishidagi integrallardir, demak,
$I = 8ln|x-1| + 3ln|x-5| + C$ bo'ladi, bu yerdagi C integraldagi o'zgarmasdir. C ning ixtiyoriyligini e'tiborga olsak, C ni lnK ga almashtirishimiz mumkin, chunki algebradan ma'lumki, $K > 0$ ligidan lnK ham ixtiyoriy o'zgarmas bo'ladi. Bu o'zgartirishlar yechimdagi barcha ifodalar logarifmik ifodalar bo'lgani uchun qilindi.

This results in the solution being expressed in the form

$$I = ln|K\,(x-1)^8(x-5)^3|.$$

Bu natijalar yechimda quyidagi ko'rinishda o'z ifodasini topadi:

$$I = ln|K\,(x-1)^8(x-5)^3|$$

Mustaqil ishlash uchun topshiriqlar.

$$\int (4u^3 - 6u^2 - 4u + 3)du;$$

$$\int (4ax^3 - 6bx^2 - 4cx + e)dx;$$

$$\int 3(2x^2 - 1)^2 \, dx$$

$$\int \frac{du}{\sqrt[3]{u^2}};$$

$$\int \frac{x^2 dx}{x^3 + 1};$$

$$\int \sin 2x \, dx.$$